Problem Set 14.2: Special Relativity Calculations

1. $t = 2t_0$

$t = \gamma t_0$

$\therefore \gamma = 2$

$\therefore \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}$

$1-\frac{v^2}{c^2} = \frac{1}{4}$

$\frac{v^2}{c^2} = \frac{3}{4} = 0.75$

$v^2 = 0.75\ (3 \times 10^8)^2$

$v = \sqrt{0.75(3\times10^8)^2}$

$= 2.60 \times 10^8\ ms^{-1}$

2. $l = 0.7\ l_0$

$l = \frac{l_0}{\gamma}$

$\therefore \gamma = \frac{1}{0.7}$

$\therefore \sqrt{1-\frac{v^2}{c^2}} = 0.7$

$1-\frac{v^2}{c^2} = (0.7)^2$

$1-\frac{v^2}{c^2} = 0.49$

$\frac{v^2}{c^2} = 0.51$

$v^2 = 0.51\ (3 \times 10^8)^2$

$v = \sqrt{0.51\ (3\times10^8)^2}$

$v = 2.14 \times 10^8\ ms^{-1}$

3. $m_o = 2.5$ t

$V = 0.92c$

$p = \gamma mv$

$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{2.5\times10^3 \times 0.92(3\times10^8)}{\sqrt{1-0.92^2}}$

$= \frac{6.75 \times 10^{11}}{\sqrt{0.1\cancel{9}536}}$

$= \cancel{1.55} \times 10^{12}\ kgms^{-1}$

$= 1.76 \times 10^{12}$

4. $u' = 0.8c$

$u = \frac{v + u'}{1+\frac{vu'}{c^2}}$

$= \frac{(0.8+0.8)c}{1+0.64}$

$= \frac{1.6c}{1.64}$

$= 0.975c$

5. $t_0 = 1\ s$

$v = 0.92c$

$t = \gamma\ t0$

$t_0 = \frac{t}{\gamma}$

$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{1}{\sqrt{1-0.92^2}}$

$= 2.55\ s$

6. $l_0 = 200$ m

$l = 160$ m

$l = \frac{l_0}{\gamma}$

$l = l_0\sqrt{1-\frac{v^2}{c^2}}$

$(\frac{l}{l_0})^2 = 1-\frac{v^2}{c^2}$

$(\frac{160^2}{200^2}) = 1-\frac{v^2}{c^2}$

$\frac{v^2}{c^2} = 1-0.8^2$

$v^2 = 0.36c^2$

v=0.6c

7a. v=0.995c

$l_0 = 250$ light years

$l = \frac{l_0}{\gamma}$

$l = l_0\sqrt{1-\frac{v^2}{c^2}}$

$= 250\sqrt{1-0.995^2}$

= 24.97 yr

$t = \frac{l}{v}$

$= \frac{24.97}{0.995}$ yr

= 25.1 yr

7b v=0.995v

l =250 light year

$t = \frac{l}{v}$

8. v=0.994c

$l_0 = 250$ light years

t=3.00 x 10^{-8} s

$l = \upsilon_0 t$

=0.994 x 3.00 x 10^8 x 3.00 x 10^{-8}

=8.946 m

$l = l_0\sqrt{1-\frac{v^2}{c^2}}$

$l = l_0\sqrt{1-\frac{v^2}{c^2}}$

$8.946 = l_0\sqrt{1-0.992^2}$

$l_0 = \frac{8.946}{0.1094}$

=81.8 m

∴ hanger length 81.8 m

9. $t_{\frac{1}{2}}(\mu) = 26\ \mu s$

$\upsilon_0 = 0.95c$

$t = t_0\sqrt{1-\frac{v^2}{c^2}}$

$= \frac{26 x 10^{-6}}{\sqrt{1-0.95^2}}$

= 8.33 x 10^{-5} s

10. $\upsilon_E = 0.70c$

$\upsilon_D = 0.85c$

a. Earth observer would see them as 0.7c + 0.85c

b. Discovery would see Earth as moving away at 0.85c

c. Wrong because the speed of light is absolute.

$$u' = \frac{u+v}{1-\frac{uv}{c^2}}$$

$$= \frac{(0.85+0.70)c}{1+0.85x0.7}$$

$$= = \frac{1.55}{1.615}c \quad 1.595$$

=0.960c ✓ 0.972c

11. a) $l = 1.2 \text{ x } 10^5$ light year

$\upsilon_0 = 0.98c$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$= 1.2 \text{ x } 10^5\sqrt{1\text{-}0.98^2}$$

= 2.39 x 10^4 light years

$t = \frac{s}{v} = \frac{2.39 \times 10^4}{0.98} = 2.44 \times 10^4$ yrs

b) $t = \frac{l}{v}$

$$= \frac{1.2 \text{ x } 10^5}{0.98}$$

=1.22 x 10^5 Yr

12. v=0.95c

$t_0 = 3$ hours

Start 0900

Finish 1200

He would observe 1200 on his clock because time is only dilated relative to the Earth observer

13. $l_0 = 600$ m

$\upsilon_0 = 0.80c$

a. they are both inertial frames provided they are moving at a constnt velocity.

b. $t = \frac{s}{v} = \frac{600}{3x10^8}$

= 2.00 x 10^{-6} s

c. distance laser travels = length of fighter as seen from mothership plus distance spaceship travels in that time

$$c\Delta t = \gamma l_0 + \upsilon\Delta t$$

$$\Delta t = \frac{\gamma l_0}{c-\upsilon} = \frac{600\sqrt{1-0.64}}{c-0.8c}$$

$$= \frac{461.0}{0.2 \text{ x } 3x10^8}$$

= 6 x 10^{-6} s

14.

15. $\upsilon_p = 3.5 \text{ x } 330 \text{ ms}^{-1}$= 1155 ms^{-1}

$\upsilon_m = 700 \text{ ms}^{-1}$

a. $\upsilon_{missile} = 1155 + 700 = 1855 \text{ ms}^{-1}$

b. $u = \frac{v-u'}{1+\frac{vu'}{c^2}}$

$$= \frac{1855}{1+\frac{1155x700}{(3x10^8)^2}}$$

$$= \frac{1855}{:1}$$

=1855 ms^{-1}

The same because not relativistic

16. $\upsilon = 0.25c$

$u' = 0.65\ c$

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$$= \frac{0.90c}{1 + 0.25 \times 0.65}$$

$$= \frac{0.9c}{1.1625}$$

$= \cancel{0.554}c\ ms^{-1}$ 0.774

17. $m_0 = 3400$ kg

$\upsilon = 2.25 \times 10^8\ ms^{-1}$

$m = \gamma m_0$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{3400}{\sqrt{1 - (\frac{2.25 \times 10^8}{3 \times 10^8})^2}}$$

$$= \frac{3400}{0.6614}$$

$= 5.14 \times 10^3$ kg

18. $l_0 = 158$ m

$\upsilon = 0.925c$

$m_o = 8.00 \times 10^5$ kg

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{8.00 \times 10^5}{\sqrt{1 - 0.925^2}}$$

6

$= 2.11 \times 10^{8}$ kg

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 158\sqrt{1 - 0.925^2}$$

$= 60$ m

19. $m = 3m_o$

$m = \gamma m_0$

$$m_0 = \frac{m}{\gamma}$$

$$m_0 = 3m_0\sqrt{1 - \frac{\upsilon^2}{c^2}}$$

$$\sqrt{1 - \frac{\upsilon^2}{c^2}} = \frac{1}{3}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{\upsilon^2}{c^2} = \frac{8}{9}$$

$$\upsilon^2 = \frac{8}{9}c^2$$

$\upsilon = 0.943c$

Problem Set 15.1: The standard model

1. The hundreds of known particles are all made from: 6 quarks, 6 leptons, 6 antiquarks, 6 antileptons, and the force carriers.
2. Neutrons and protons are made up of quarks, which are held together by gluons
   Electrons are fundamental particles and are classified as leptons.
3. Baryons and mesons are hadrons.

   BARYONS - are any hadron made of three quarks (qqq). Protons and neutrons are baryons because they are each made of three quarks – protons two up and one down quark (uud) and neutrons one up and two down (udd).

   MESONS are hadrons made from a quark and its anti-quark (eg pion or pi-meson). One example of a meson is a pion (+), which is made of an up quark and a down anitiquark. The antiparticle of a meson just has its quark and antiquark switched, so an antipion (-) is made of a down quark and an up antiquark. Because a meson consists of a particle and an antiparticle, it is very unstable. The K meson lives much longer than most mesons, which is why it was called "strange" and gave this name to the strange quark, one of its components.
4. Both muon and antimuon have a mass of 105.66 MeV/c^2. (Assuming the particles are slow moving)

   (a) The two photons will each have energies of 105.66MeV.

   (b) Using $E = h\nu = hc/\lambda$

   $\lambda = hc/E =$ 6.62 x 10^{-34} x 3x10^8/(105.66 x 10^6 x 1.6x10^{-19}) = 1.18×10^{-14} m

   (c) Two photons are required to conserve momentum

   (d) They must travel in opposite directions to conserve momentum

   (e) The photons are in the gamma radiation part of the e/m spectrum.
5. (a) n → p + e^- + _____

	CHARGE	BARYON No.	LEPTON No.
LHS	0	1	0
RHS	0	1	1
Balance	0	0	-1

An anti-neutrino is required on the RHS to balance

(b) ___ + n → ____ + e^-

	CHARGE	BARYON No.	LEPTON No.
LHS	0	1	0
RHS	-1	0	1
Balance	1	+1	+1

electron ~~anti~~neutrino(LHS) and proton (RHS)

(c) π^+ → μ^+ + ____

	CHARGE	BARYON No.	LEPTON No.

LHS	1	0	0
RHS	1	0	-1
Balance	0	0	+ -1

A muon ~~anti~~neutrino is given off.

(d) p → n + v_e + ____

	CHARGE	BARYON No.	LEPTON No.
LHS	~~0~~ +1	1	~~1~~ 0
RHS	~~1~~ 0	1	~~0~~ 0 +1
Balance	+1	0	-1

An ~~antimuon~~ e^+ positron

6. Each γ–ray must have energy of 511keV (the rest energy of an electron and positron). To conserve momentum two γ–rays are produced travelling in opposite directions.
7. Positrons will travel anti-clockwise
8. Yes an object can accelerate while keeping the same speed, it they are undergoing circular motion at constant angular velocity.
9. Conservation of momentum is violated – the particles have momentum in the y-direction (toward top of page) which they didn't possess before the collision.
10. (a) Friction is caused by residual electromagnetic interactions between the atoms of the two materials. The force carriers are photons and (W and Z bosons).?

    (b) Nuclear bonding is caused by residual strong interactions between the various parts of the nucleus. The force carriers are gluons.

    (c) The planets orbits due to gravitons.
11. (a) Weak and Gravity interactions act on neutrinos

    (b) Weak (W+, W-, and Z) interactions have heavy carriers

    (c) All of interactions act on the protons in you
12. Gluons cannot be isolated because they carry colour charge themselves.
13. Gravitons are hypothetical particles to explain the 'force' of gravity. They have not been observed. (Gluons have been observed indirectly.)

Problem Set 15.2: General revision questions

1. Using $\lambda = \dfrac{hc}{pc}$

   Where $pc = \sqrt{2.E_K.m_0c^2}$
   Here $E_K = 7.0 \times 10^{12}$ eV
   $pc = 1.1469 \times 10^{11}$ eV
   $m_0c^2 = 511 \times 10^3$ eV
   $hc = 1239.84$ eV.nm (this is a constant)
   $l = 1.08 \times 10^{-17}$ m

1. $\lambda = \dfrac{h}{p}$ $\quad p = \dfrac{mv}{\sqrt{1-v^2/c^2}}$

$= \dfrac{6.63\times10^{-34}}{\left(\dfrac{1.67\times10^{-27} \times 0.999997 \times 3\times10^8}{\sqrt{1-0.999997^2}}\right)}$

$= 3.24 \times 10^{-18}$ m

2. Using $\dfrac{v}{c} \approx 1 - \frac{1}{2}\left(\dfrac{m_0c^2}{E_{tot}}\right)^2$ for $v \approx c$

   Where $E_{tot} \approx E_K = 3.00 \times 10^9$ eV and $m_0c^2 = 5.11 \times 10^5$ eV
   $v = 0.999999985493c$

2.a) $E = \dfrac{mc^2}{\sqrt{1-v^2/c^2}}$

mc^2 + $3.000\times10^9 \times 1.6\times10^{-19} = \dfrac{9.11\times10^{-31} \times (3\times10^8)^2}{\sqrt{1-v^2/c^2}}$

4.8008 $\times10^{-10} = \dfrac{8.199\times10^{-4}}{\sqrt{1-v^2/c^2}}$

$\sqrt{1-v^2/c^2} = 1.708\times10^{-4}$

$1-v^2/c^2 = 2.9\times10^{-8}$

$v^2/c^2 = 0.9999997$

$v/c = 0.999999985$

$V = 0.999999985c$

b) $\lambda = \dfrac{h}{p}$ $\quad p = \dfrac{mv}{\sqrt{1-v^2/c^2}} = 1.60\times10^{-18}$ kgms^{-1}

$\lambda = 4.143\times10^{-16}$ m

c) $m = \dfrac{m_0}{\sqrt{1-v^2/c^2}} = 5.33\times10^{-27}$ kg

3. Using $m_{Rel} = g.m_0$ where $g = 707.1$ and $m_0 = 1.67 \times 10^{-27}$ kg
   $m_{Rel} = 1.18 \times 10^{-24}$ kg.

4. $E = 7.53 \times 10^{-13}$ J
   $m = E/c^2$
   Mass $= 8.38 \times 10^{-30}$ kg

5. (a). $m_ec^2 = 8.19 \times 10^{-14}$ J, 5.11×10^5 eV
   (b). $gm_ec^2 = 3.12 \times 10^{-13}$ J, 1.95×10^6 eV
   (c). $E_K = (g-1)\, m_ec^2 = 2.303 \times 10^{-13}$ J, 1.44×10^6 eV

6. Using: $\dfrac{v}{c} \approx \sqrt{1-\left(\dfrac{m_0c^2}{E_{tot}}\right)^2}$

   $E_{tot} = E_k + m_ec^2$
   $E_K = 40{,}000$ eV
   (a). $v_{max} = 0.374c$
   (b). 40,000 eV

a) $E_{TOT} = 1.6\times10^{-19} \times 40\,000 + m_0c^2$

$E_{TOT} = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} = 8.839\times10^{-14}$

$1-v^2/c^2 = 0.86043$

$v = 0.374c$

b) 40 000 eV

7. No energy is released – 605 MeV is required to make this reaction occur:
   (139.6 + 938.3) - (1189.4 + 493.7) = -605MeV

8. Mass Pa236 = 236.04868 u
   Mass U236 = 236.045568 u
   Mass difference = 0.003112 u
   $Dmc^2 = = 2.9$ MeV
   KE of recoil nucleus = approx. 33eV which is negligible
   E beta = 2.9 MeV
   V beta = 0.989c

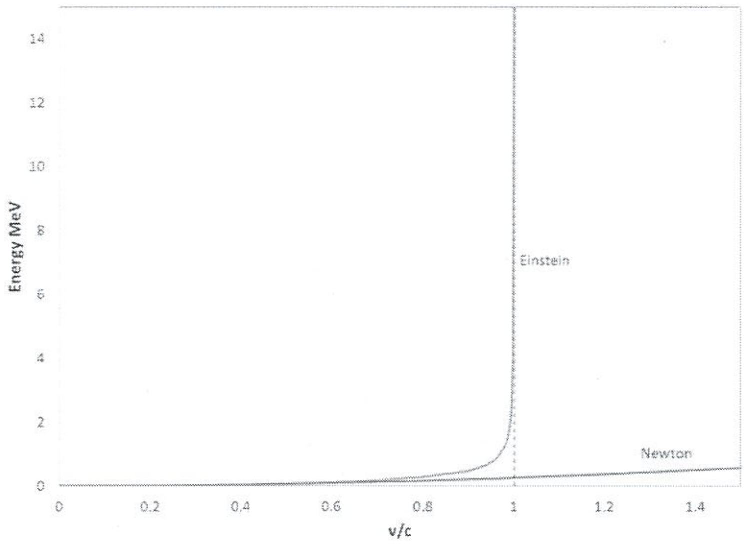
a) $E_{TOT} = KE + m_0c^2$

$v =$ 0.989c

9. g = 707.1 $\qquad m_{rel} = 1.18 \times 10^{-24}$ kg

10. 2.16×10^{-23} kg $\qquad$ 0.00209 km

11. In the Synchrotron electrons are accelerated to velocities approaching the velocity of light. The graphs of both the non-relativistic energy and the relativistic energy are shown



12. (a). 0.511 MeV
    (b). $p_{rel} = 2.05 \times 10^{-14}$ kg.m/s' $p_{classical} = 2.73 \times 10^{-22}$ kg.m/s $p_{rel} = 1.12 \times 10^{-17}$ kgms^{-1}
    Ratio = 41 000 × heavier
    (c). 6.15×10^{-06} J, 3.84×10^{13} eV × 3.34×10^{-9} J

13. See problem 2.

14. It is moving away (red shifted) 17.a) $\Sigma p = 0$

    Using: $\frac{v}{c} = \frac{\left(\frac{\lambda_0}{\lambda}\right)^2 - 1}{\left(\frac{\lambda_0}{\lambda}\right)^2 + 1}$

    research station b) $p = \frac{mv}{\sqrt{1 - v^2/c^2}} = 2.67 \times 10^{22}$ kgms^{-1} for 1e$^-$
    For both $p_{total} = 0$

    v = 0.72c (moving apart)

    $E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = 1.15 \times 10^{-13}$ J

15. Wavelength green light = 540nm (±30nm)
    Wavelength red light = 700nm (±30nm)
    v = 0.25c toward the light

    electron: $v = 0$ $p = 0$ (for electron)

16. Relativistic mass $= 2.00 \times 10^{-30}$ kg
    KE: 9.77×10^{-14} J, 6.10×10^{5} eV

18 ~~17~~. Gamma x 26 ms = 3.2 x 26 ms = 83 μ~~m~~s

19. See 18 – note here the half-life is stated at 260 ms not 26 ms.
    (a) They will appear to have a half-life of ~~830 ms~~. 833μs = 8.33×10^{-8} s
    (b) distance travelled = 0.95 x c m/s x 830 x 10^{-9} s = 236m 23.7m
    (c). 74m 7.41m

20. (a). Toward the Earth
    (b). 0.2c 0.18c
    (c). Apart
    (d) 0.24c